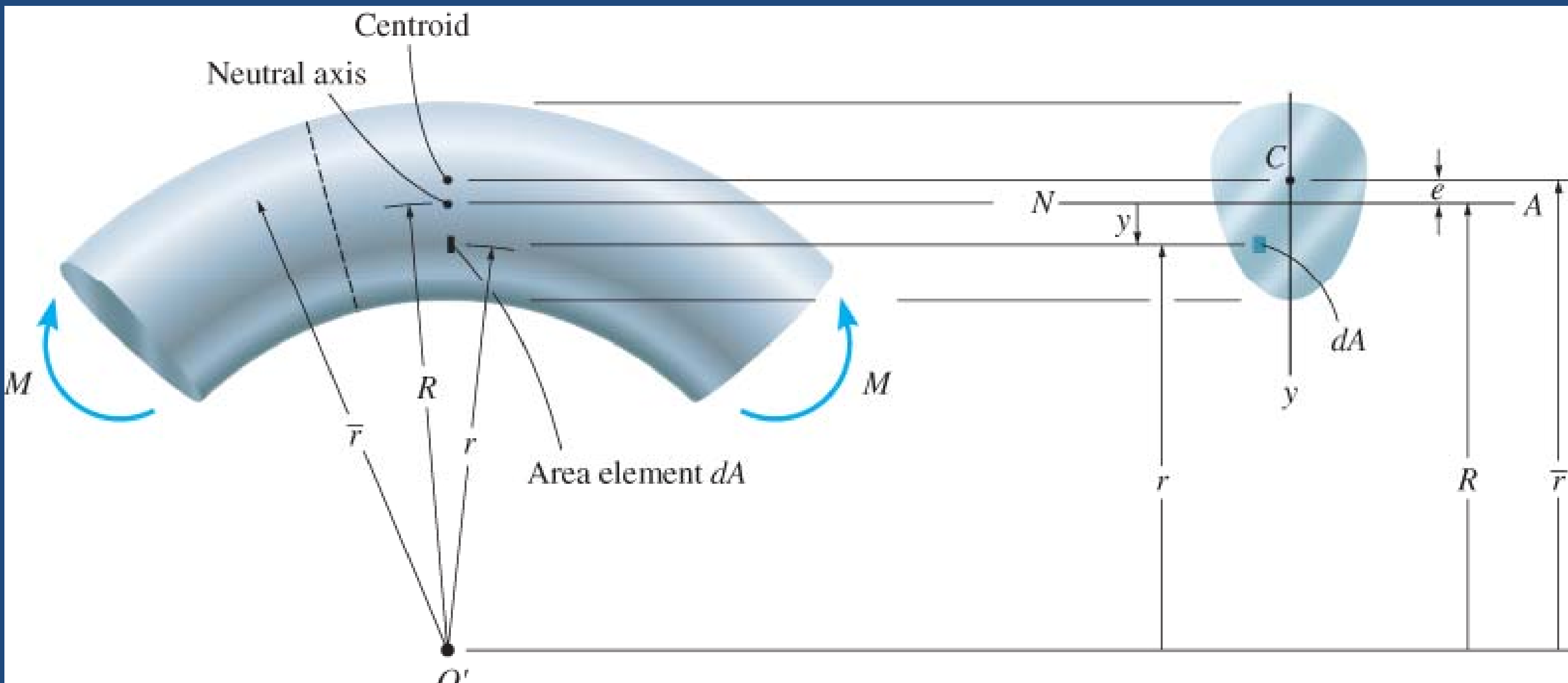
CURVED BEAMS

- Flexure formula only applies to members that are straight as normal strain varies linearly from the neutral axis
- Thus another equation needs to be formulated for curved beam, i.e., a member that has a curved axis and is subjected to bending

- Assumptions for analysis:
  1. X-sectional area is constant and has an axis of symmetry that is perpendicular to direction of applied moment **M**
  2. Material is homogeneous and isotropic and behaves in linear-elastic manner under loading
  3. X-sections of member *remain plane* after moment applied and distortion of x-section within its own will be neglected

- By first principles:



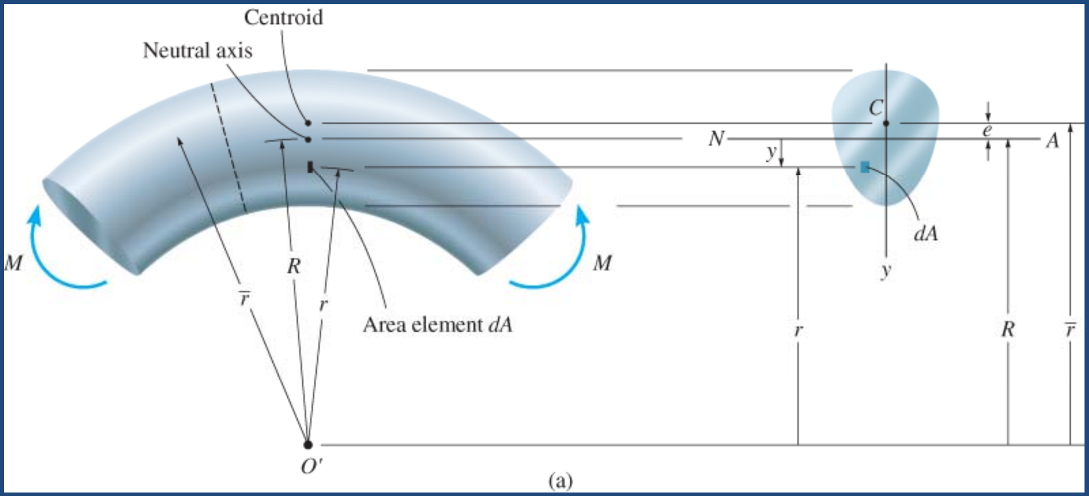
(a)

$$\sigma = Ek\,(R\text{-}r/r)$$

Location of neutral axis:

$$R = \frac{A}{\int_A \frac{dA}{r}}$$

Equation 6-23



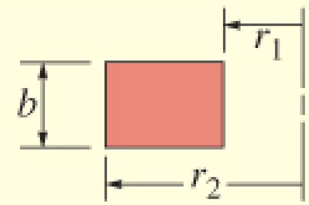
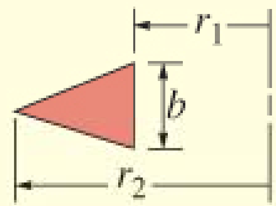
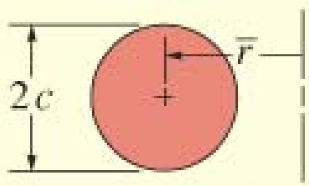
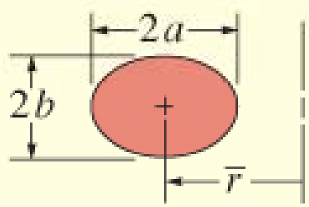
R = location of neutral axis, specified from center of curvature O' of member

A = x-sectional area of the member

R = arbitrary position of the area element dA on x-section specified from center of curvature P' of member

Common x-sections to use in integral in Eqn 6-23

$$R = \frac{A}{\int_A \frac{dA}{r}}$$

Shape	Area	$\int_A \frac{dA}{r}$
	$b(r_2 - r_1)$	$b \ln \frac{r_2}{r_1}$
	$\frac{b}{2}(r_2 - r_1)$	$\frac{b\, r_2}{(r_2 - r_1)}\left(\ln \frac{r_2}{r_1}\right) - b$
	πc^2	$2\pi\left(\bar{r} - \sqrt{\bar{r}^2 - c^2}\right)$
	πab	$\frac{2\pi b}{a}\left(\bar{r} - \sqrt{\bar{r}^2 - a^2}\right)$

Normal stress in curved beam:

$$\sigma = \frac{M(R-r)}{Ar(\bar{r}-R)}$$

Equation 6-24

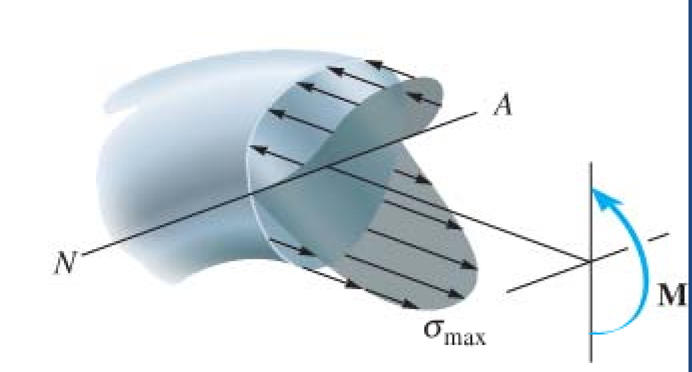
$$\sigma = \frac{My}{Ae(R-y)}$$

Equation 6-25

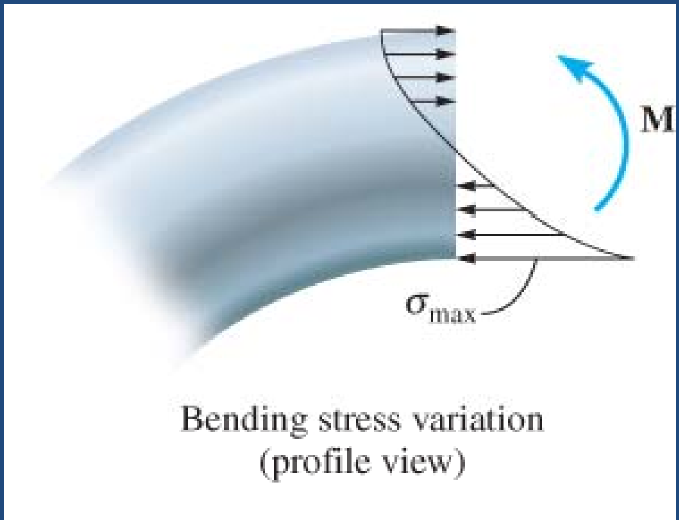
- The above equations represent 2 forms of the curved-beam formula, used to determine the normal-stress distribution in a member

Normal stress in curved beam:

- The stress distribution is as shown, hyperbolic, and is sometimes called circumferential stress
- Radial stress will also be created as a result
- If radius of curvature is greater than 5 times the depth of member, flexure formula can be used to determine the stress instead



(d)



(c)

IMPORTANT

- Curved beam formula used to determine circumferential stress in a beam when radius of curvature is less than five times the depth of the beam
- Due to beam curvature, normal strain in beam does not vary linearly with depth as in the case of straight beam. Thus, neutral axis does not pass through centroid of section
- Ignore radial stress component of bending, if x-section is a solid section and not made from thin plates

Procedure for analysis

Section properties

- Determine x-sectional area A and location of centroid r, measured from centre of curvature
- Compute location of neutral axis, R using Eqn 6-23 or Table 6-2. If x-sectional area consists of n "composite" parts, compute $\int dA/r$ for each part.
- From Eqn 6-23, for entire section,
- In all cases, $R < r$

$$R = \frac{A}{\int_A \frac{dA}{r}}$$

Procedure for analysis

Normal stress

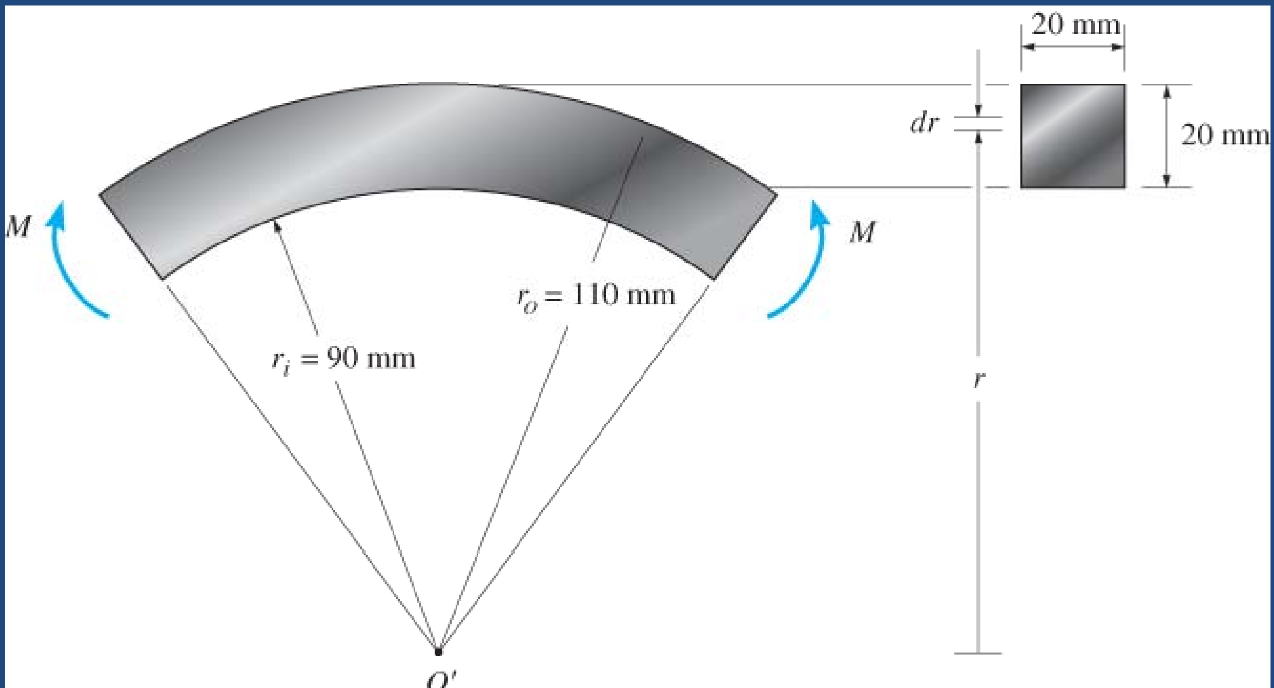
- Normal stress located at a pt r away from the centre of curvature is determined Eqn 6-24. If distance y to pt is measured from neutral axis, then compute $e = \overline{r} - R$ and use Eqn 6-25
- Since $\overline{r} - R$ generally produces a very small number, it is best to calculate r and R with sufficient capacity so that subtraction leads to e with at least 3 significant figures

Procedure for analysis

Normal stress

- Positive stress indicates tensile stress, negative means compressive stress
- Stress distribution over entire x-section can be graphed, or a volume element of material can be isolated and used to represent stress acting at the pt on x-section where it has been calculated

Steel bar with rectangular x-section is shaped into a circular arc. Allowable normal stress is σ_{allow} = 140 MPa. Determine maximum bending moment **M** that can be applied to the bar. What would this moment be if the bar was straight?



(a)

Internal moment

Since **M** tends to increase bar's radius of curvature, it is positive.

Section properties

Location of neutral axis is determined using Eqn 6-23.

$$\int_A \frac{dA}{r} = \int_{90\text{ mm}}^{110\text{ mm}} \frac{(20\text{ mm})\,dr}{r} = 4.0134\text{ mm}$$

Similar result can also be obtained from Table 6-2.

Section properties

We do not know if normal stress reaches its maximum at the top or bottom of the bar, so both cases must be compute separately.

Since normal stress at bar top is $\sigma = -140$ MPa

$$\sigma = \frac{M(R - r_o)}{Ar_o(\bar{r} - R)}$$

$$\vdots$$

$$M = 0.199 \text{ kN}\cdot\text{m}$$

Section properties

Likewise, at bottom of bar, $\sigma = +140$ MPa

$$\sigma = \frac{M(R - r_i)}{A r_i (r - R)}$$

$$\vdots$$

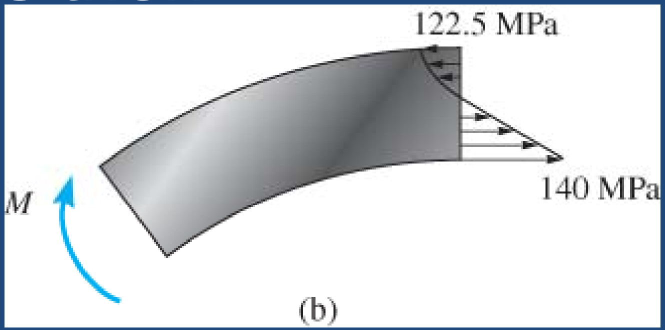
$$M = 0.174 \text{ kN·m}$$

By comparison, maximum that can be applied is 0.174 kN·m, so maximum normal stress occurs at bottom of the bar.

Section properties

Compressive stress at top of bar is then

$$\sigma = 122.5 \text{ N/mm}^2$$



By comparison, maximum that can be applied is 0.174 kN·m, so maximum normal stress occurs at bottom of the bar.

If bar was straight?

$$\sigma = Mc/I$$

$$\vdots$$

$$M = 0.187 \text{ kN}\cdot\text{m}$$

This represents an error of about 7% from the more exact value determined above.